Chapter 13

Determinants

Exercise

1. $\begin{vmatrix} \sin^2 x & \cos^2 x & 1 \\ \cos^2 x & \sin^2 x & 1 \\ -10 & 12 & 2 \end{vmatrix}$ is equal to

(a) 0
(b) $12\cos^2 x - 10\sin^2 x$
(c) $12\sin^2 x - 10\cos^2 x - 2$
(d) $10\sin^2 x$

2. $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix}$ equals to

(a) $x+y$ (b) xy
(c) $x-y$ (d) $1+x+y$

3. Let $A = \begin{vmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{vmatrix}$

If $|A^2| = 25$, then $|\alpha|$ equals to

(a) 5^2 (b) 1
(c) 1/5 (d) 5

4. If a, b, c are in AP, then the value of

$\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix}$ is

(a) 3 (b) – 3
(c) 0 (d) None of these

5. The value of the determinant

$\begin{vmatrix} 1 & 1 & 1 \\ {}^{m}C_1 & {}^{m+1}C_1 & {}^{m+2}C_1 \\ {}^{m}C_2 & {}^{m+1}C_2 & {}^{m+2}C_2 \end{vmatrix}$ is equal to

(a) 1 (b) – 1
(c) 0 (d) None of these

6. If α, β, γ are the roots of $x^3 + ax^2 + b = 0$ then the value of $\begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix}$ is

(a) $-a^3$ (b) $a^3 - 3b$
(c) a^3 (d) $a^2 - 3b$

7. The determinant

$\Delta = \begin{vmatrix} a & b & ax+b \\ b & c & bx+c \\ ax+b & bx+c & 0 \end{vmatrix}$ is equal to zero, if

(a) a, b, c are in AP
(b) a, b, c are in GP
(c) a, b, c are in HP
(d) α is a root of $ax^2 + bx + c = 0$

8. If x, y, z are all distinct and $\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$ then the value of xyz is

(a) – 2 (b) – 1
(c) – 3 (d) None of these

9. If $\begin{vmatrix} b+c & c+a & a+b \\ a+b & b+c & c+a \\ c+a & a+b & b+c \end{vmatrix} = k\begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}$, then the value of k is

(a) 1 (b) 2
(c) 3 (d) 4

10. If $a^{-1} + b^{-1} + c^{-1} = 0$ such that $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = \lambda$ then the value of λ is

(a) 0 (b) abc
(c) $-abc$ (d) None of these

11. The value of λ and μ for which the system of equations $x + y + z = 6$, $x + 2y + 3z = 10$ and $x + 2y + \lambda z = \mu$ has a unique solution are
(a) $\lambda \neq 3, \mu \in R$ (b) $\lambda = 3, \mu = 10$
(c) $\lambda \neq 3, \mu = 10$ (d) $\lambda \neq 3, \mu \neq 10$

12. The value of λ and μ for which the system of equations $x + y + z = 6$, $x + 2y + 3z = 10$ and $x + 2y + \lambda z = \mu$ has no solution are
(a) $\lambda = 3, \mu = 10$ (b) $\lambda = 3, \mu \neq 10$
(c) $\lambda \neq 3, \mu = 10$ (d) $\lambda \neq 3, \mu \neq 10$

13. The value of λ and μ for which the system of equations $x + y + z = 6$, $x + 2y + 3z = 10$ and $x + 2y + \lambda z = \mu$ has infinite number of solutions are
(a) $\lambda = 3, \mu = 10$ (b) $\lambda = 3, \mu \neq 10$
(c) $\lambda \neq 3, \mu = 10$ (d) $\lambda \neq 3, \mu \neq 10$

14. If a, b, c are non-zero real numbers, then $\begin{vmatrix} bc & ca & ab \\ ca & ab & bc \\ ab & bc & ca \end{vmatrix}$ vanishes, when
(a) $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=0$ (b) $\frac{1}{a}-\frac{1}{b}-\frac{1}{c}=0$
(c) $\frac{1}{b}+\frac{1}{c}-\frac{1}{a}=0$ (d) $\frac{1}{b}-\frac{1}{c}-\frac{1}{a}=0$

15. The value of the determinant $\begin{vmatrix} ka & k^2+a^2 & 1 \\ kb & k^2+b^2 & 1 \\ kc & k^2+c^2 & 1 \end{vmatrix}$ is
(a) $k(a+b)(b+c)(c+a)$
(b) $k\,abc(a^2+b^2+c^2)$
(c) $k(a-b)(b-c)(c-a)$
(d) $k(a+b-c)(b+c-a)(c+a-b)$

16. The system of simultaneous equations $kx + 2y - z = 1$, $(k-1)y - 2z = 2$ and $(k+2)z = 3$ have a unique solution if k equals
(a) -2 (b) -1
(c) 0 (d) 1

17. The value of the determinant $\begin{vmatrix} 1 & \omega^3 & \omega^5 \\ \omega^3 & 1 & \omega^4 \\ \omega^5 & \omega^4 & 1 \end{vmatrix}$, where ω is an imaginary cube root of unity, is
(a) $(1-\omega)^2$ (b) 3
(c) -3 (d) None of these

18. If the system of equations $x + ay + az = 0$; $bx + y + bz = 0$ and $cx + cy + z = 0$ where a, b and c are non-zero non unity, has a non-trivial solution, then the value of $\frac{a}{1-a}+\frac{b}{1-b}+\frac{c}{1-c}$ is
(a) 0 (b) 1
(c) -1 (d) $\frac{abc}{a^2+b^2+c^2}$

19. If $\Delta_1 = \begin{vmatrix} x & b & b \\ a & x & b \\ a & a & x \end{vmatrix}$ and $\Delta_2 = \begin{vmatrix} x & b \\ a & x \end{vmatrix}$, then
(a) $\Delta_1 = 3(\Delta_2)^2$ (b) $\frac{d}{dx}(\Delta_1) = 3\Delta_2$
(c) $\frac{d}{dx}(\Delta_1) = 3\Delta_2^2$ (d) $\Delta_1 = 3(\Delta_2)^{3/2}$

20. If $a \neq p$, $b \neq q$, $c \neq r$ and $\begin{vmatrix} p & b & c \\ a & q & c \\ a & b & r \end{vmatrix} = 0$, then the value of $\frac{p}{p-a}+\frac{q}{q-b}+\frac{r}{r-c}$ is
(a) 0 (b) 1
(c) -1 (d) 2

21. The factors of $\begin{vmatrix} x & a & b \\ a & x & b \\ a & b & x \end{vmatrix}$ are
(a) $x-a$, $x-b$ and $x+a+b$
(b) $x+a$, $x+b$ and $x+a+b$
(c) $x+a$, $x+b$ and $x-a-b$
(d) $x-a$, $x-b$ and $x-a-b$

22. If ω is a cube root of unity, then $\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix}$ is equal to
(a) 1 (b) ω
(c) ω^2 (d) 0

23. Value of $\begin{vmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 1 & \log_y z \\ \log_z x & \log_z y & 1 \end{vmatrix}$ where x, y, z are positive, is
(a) 1 (b) 0
(c) $\log_y x$ (d) $\log_z y$

24. The roots of the equations $\begin{vmatrix} x-1 & 1 & 1 \\ 1 & x-1 & 1 \\ 1 & 1 & x-1 \end{vmatrix} = 0$ are
(a) 1, 2 (b) -1, 2
(c) 1, -2 (d) -1, -2

25. If $U_n = \begin{vmatrix} n & 15 & 8 \\ n^2 & 35 & 9 \\ n^3 & 25 & 10 \end{vmatrix}$ then $\sum_{n=1}^{5} U_n$ is equal to
(a) 0 (b) 25
(c) 625 (d) None of these

26. If k is a scalar and A is an $n \times n$ square matrix. Then $|kA|$ is equal to
(a) $k\,|A|^n$ (b) $k\,|A|$
(c) $k^n\,|A^n|$ (d) $k^n\,|A|$

27. $\begin{vmatrix} 11 & 12 & 13 \\ 12 & 13 & 14 \\ 13 & 14 & 15 \end{vmatrix}$ is equal to

(a) 1 (b) 0
(c) – 1 (d) 67

28. If every element of a third order determinant of value Δ is multiplied by 5, then the value of new determinant is
(a) Δ (b) 5Δ
(c) 25Δ (d) 125Δ

29. A root of the equation $\begin{vmatrix} 3-x & -6 & 3 \\ -6 & 3-x & 3 \\ 3 & 3 & -6-x \end{vmatrix} = 0$ is
(a) 6 (b) 3
(c) 0 (d) None of these

30. The system of linear equations $x + y + z = 2$, $2x + y - z = 3$, $3x + 2y + kz = 4$ has a unique solution if
(a) $k \neq 0$ (b) $-1 < k < 1$
(c) $-2 < k < 2$ (d) $k = 0$

31. If $\frac{a_1}{x} + \frac{b_1}{y} = c_1, \frac{a_2}{x} + \frac{b_2}{y} = c_2$ and

$$\Delta_1 = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}, \Delta_2 = \begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}, \Delta_3 = \begin{vmatrix} c_1 & a_1 \\ c_2 & a_2 \end{vmatrix}$$

Then (x, y) is equal to which one of the following ?
(a) $\left(\frac{\Delta_2}{\Delta_1}, \frac{\Delta_3}{\Delta_1}\right)$ (b) $\left(\frac{\Delta_3}{\Delta_1}, \frac{\Delta_2}{\Delta_1}\right)$
(c) $\left(\frac{\Delta_1}{\Delta_2}, \frac{\Delta_1}{\Delta_3}\right)$ (d) $\left(\frac{-\Delta_1}{\Delta_2}, \frac{-\Delta_1}{\Delta_3}\right)$

32. What is the value of $\begin{vmatrix} 1-i & \omega^2 & \omega \\ \omega^2 + i & \omega & -i \\ 1-2i-\omega^2 & \omega^2 - \omega & i+\omega \end{vmatrix}$, where ω is the cube roots of unity ?
(a) – 1 (b) 1
(c) 2 (d) 0

33. If $\Delta = \begin{vmatrix} 1 & bc & bc(b+c) \\ 1 & ca & ca(c+a) \\ 1 & ab & ab(a+b) \end{vmatrix}$, then its value is equal to
(a) $a^3 + b^3 + c^3 - 3abc$
(b) $abc(a + b + c)$
(c) independent of a, b, c
(d) zero

34. If $\Delta = \begin{vmatrix} i^n & i^{n+1} & i^{n+2} \\ i^{n+5} & i^{n+4} & i^{n+3} \\ i^{n+6} & i^{n+7} & i^{n+8} \end{vmatrix}$, where $i = \sqrt{-1}$, then its values is
(a) $0 \ \forall \ n \in R$ (b) 1 if $n = 4k$
(c) $-i$ if $n = 3k$ (d) None of these

35. If $\Delta_1 = \begin{vmatrix} a & b & 2c \\ p & q & 2r \\ x & y & 2z \end{vmatrix}$ and $\Delta_2 = \begin{vmatrix} r & 2p & q \\ 2z & 4x & 2y \\ c & 2a & 2b \end{vmatrix}$ then Δ_1/Δ_2 is equal to
(a) 1 (b) 2
(c) – 1 (d) 1/2

36. The value of the determinant with out expansion $\begin{vmatrix} b^2-ab & b-c & bc-ac \\ ab-a^2 & a-b & b^2-ab \\ bc-ac & c-a & ab-a^2 \end{vmatrix}$ is
(a) abc (b) $a + b + c$
(c) 0 (d) $a^2 + b^2 + c^2$

37. The value of the determinant

$$\Delta = \begin{vmatrix} \log x & \log y & \log z \\ \log 2x & \log 2y & \log 2z \\ \log 3x & \log 3y & \log 3z \end{vmatrix}$$ is

(a) 0 (b) log (xyz)
(c) log $(6xyz)$ (d) 6 log (xyz)

38. If $f(x) = \begin{vmatrix} a & -1 & 0 \\ ax & a & -1 \\ ax^2 & ax & a \end{vmatrix}$, then $f(2x) - f(x)$ is equal to
(a) ax (b) $ax(2a + 3x)$
(c) $ax(2 + 3x)$ (d) None of these

39. If $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix}$ for $x \neq 0, y \neq 0$, the D is
(a) divisible by neither x nor y
(b) divisible by both x and y
(c) divisible by x but not y
(d) divisible by y but not x

40. If ω be imaginary cube root of unity, then

$$\begin{vmatrix} \lambda+1 & \omega & \omega^2 \\ \omega & \lambda+\omega^2 & 1 \\ \omega^2 & 1 & \lambda+\omega \end{vmatrix}$$ is equal to

(a) 0 (b) $\lambda^3 + 1$
(c) λ^3 (d) None of these

41. The value of n for which the determinant

$$\Delta = \begin{vmatrix} ^8C_3 & ^9C_5 & ^{10}C_7 \\ ^8C_4 & ^9C_6 & ^{10}C_8 \\ ^9C_n & ^{10}C_{n+2} & ^{11}C_{n+4} \end{vmatrix} = 0$$ is

(a) 2 (b) 3
(c) 4 (d) None of these

42. If ω is complex cube root of unity, then $\begin{vmatrix} 1 & 1-i & -i \\ 1+i+\omega^2 & -1 & -1+\omega-i \\ \omega^2 & \omega^2-1 & -1 \end{vmatrix}$ is equal to

(a) 1 (b) ω
(c) i (d) 0

43. Let a, b, c be cube roots of unity and
$$\Delta = \begin{vmatrix} a^2+b^2 & c^2 & c^2 \\ a^2 & b^2+c^2 & a^2 \\ b^2 & b^2 & c^2+a^2 \end{vmatrix},$$ then

(a) Re(Δ) = 0
(b) lm(Δ) = 0
(c) Re(Δ) + lm(Δ) = 0
(d) Re(Δ) lm(Δ) = 4

44. The value of the determinant
$$\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix}$$ is

(a) $2(a + b + c)$ (b) $2(a + b + c)^3$
(c) $ab + bc + ca$ (d) $2bc(ab + bc + ca)$

45. Let $\Delta = \begin{vmatrix} a & a+b & a+b+c \\ 3a & 4a+3b & 5a+4b+3c \\ 6a & 9a+6b & 11a+9b+6c \end{vmatrix}$

where $a = i$, $b = \omega$, and $c = \omega^2$, then Δ equals

(a) ω (b) $-\omega^2$
(c) i (d) $-i$

46. If $\begin{vmatrix} p & q-y & r-z \\ p-x & q & r-z \\ p-x & q-y & r \end{vmatrix} = 0$ then the value of $\frac{p}{x}+\frac{q}{y}+\frac{r}{z}$ is

(a) 0 (b) 1
(c) 2 (d) 4 pqr

47. The determinant
$$\begin{vmatrix} \cos(\theta+\phi) & -\sin(\theta+\phi) & \cos(2\phi) \\ \sin\theta & \cos\theta & \sin\phi \\ -\cos\theta & \sin\theta & \cos\phi \end{vmatrix}$$ is

(a) ≠ 0
(b) independent of θ
(c) independent of ϕ
(d) independent of both θ and ϕ

48. If $\begin{vmatrix} x+a & a^2 & a^3 \\ x+b & b^2 & b^3 \\ x+c & c^2 & c^3 \end{vmatrix} = 0$, $a \neq b \neq c$, then x is equal to

(a) $\frac{ab}{\sum ab}$ (b) $-\frac{abc}{\sum ab}$
(c) $\frac{\sum ab}{abc}$ (d) $-\frac{\sum ab}{abc}$

49. $\begin{vmatrix} x^2 & x^2-(y-z)^2 & yz \\ y^2 & y^2-(z-x)^2 & xz \\ z^2 & z^2-(x-y)^2 & xy \end{vmatrix}$ is equal to

(a) $(x-y)(y-z)(z-x)\Sigma x$
(b) $(x-y)(y-z)(z-x)\Sigma xy$
(c) $(x-y)(y-z)(z-x)\Sigma x^2$
(d) $(x-y)(y-z)(z-x)(\Sigma x^2)\Sigma x$

50. The value of a for which the system of equations $a^3x + (a+1)^3 y + (a+2)^3 z = 0$, $ax + (a+1)y + (a+2)z = 0$ and $x + y + z = 0$ has a non-zero solution is

(a) 1 (b) 0
(c) – 1 (d) None of these

ANSWERS

1.	(a)	**2.**	(b)	**3.**	(c)	**4.**	(c)	**5.**	(a)	**6.**	(c)	**7.**	(b)	**8.**	(b)	**9.**	(b)	**10.**	(b)
11.	(a)	**12.**	(b)	**13.**	(a)	**14.**	(a)	**15.**	(c)	**16.**	(b)	**17.**	(b)	**18.**	(c)	**19.**	(c)	**20.**	(d)
21.	(a)	**22.**	(d)	**23.**	(b)	**24.**	(b)	**25.**	(d)	**26.**	(d)	**27.**	(b)	**28.**	(d)	**29.**	(c)	**30.**	(a)
31.	(d)	**32.**	(d)	**33.**	(c)	**34.**	(a)	**35.**	(d)	**36.**	(c)	**37.**	(a)	**38.**	(b)	**39.**	(b)	**40.**	(c)
41.	(c)	**42.**	(d)	**43.**	(b)	**44.**	(b)	**45.**	(c)	**46.**	(c)	**47.**	(b)	**48.**	(b)	**49.**	(d)	**50.**	(c)

Explanations

1. (a) $\Delta = \begin{vmatrix} \sin^2 x & \cos^2 x & 1 \\ \cos^2 x & \sin^2 x & 1 \\ -10 & 12 & 2 \end{vmatrix}$

Applying $C_3 \to C_3 - (C_1 + C_2)$

$$\Delta = \begin{vmatrix} \sin^2 x & \cos^2 x & 0 \\ \cos^2 x & \sin^2 x & 0 \\ -10 & 12 & 0 \end{vmatrix} = 0$$

2. (b) $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix}$

Applying, $R_2 \to R_2 - R_1$ and $R_3 \to R_3 \to R_1$

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 0 & x & 0 \\ 0 & 0 & y \end{vmatrix} = xy$$

3. (c) $A = \begin{vmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{vmatrix}$

$A = 25\alpha$

Given $|A^2| = 25 \Rightarrow (25\alpha)^2 = 25$

$\Rightarrow \alpha = \dfrac{1}{5}$

4. (c) a, b, c are in AP.

$\therefore 2b = a + c$

$$D = \begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix}$$

$$= \begin{vmatrix} x+1 & x+2 & x+a \\ 0 & 0 & 0 \\ x+3 & x+4 & x+c \end{vmatrix} \{R_2 \to 2R_2 - (R_1 + R_3)\}$$

$= 0$

5. (a) $\begin{vmatrix} 1 & 1 & 1 \\ {}^mC_1 & {}^{m+1}C_1 & {}^{m+2}C_1 \\ {}^mC_2 & {}^{m+1}C_2 & {}^{m+2}C_2 \end{vmatrix}$

$$= \begin{vmatrix} 1 & 1 & 1 \\ m & m+1 & m+2 \\ \dfrac{m(m-1)}{2} & \dfrac{(m+1)m}{2} & \dfrac{(m+2)(m+1)}{2} \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 0 & 0 \\ m & 1 & 2 \\ \dfrac{m(m-1)}{2} & m & 2m+1 \end{vmatrix}$$

$C_2 \to C_2 - C_1$, $C_3 \to C_3 - C_1$

$= 2m + 1 - 2m = 1$

6. (c) Since, α, β, γ are the roots of the equation

$x^3 + ax^2 + b = 0$

Therefore, $\alpha + \beta + \gamma = -a$

$\alpha\beta + \beta\gamma + \gamma\alpha = 0$ and $\alpha\beta\gamma = -b$

Now, $\begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix}$

$= -(\alpha + \beta + \gamma)(\alpha^2 + \beta^2 + \gamma^2 - \alpha\beta - \beta\gamma - \gamma\alpha)$

$= -(\alpha + \beta + \gamma)\{(\alpha + \beta + \gamma)^2 - 3(\alpha\beta + \beta\gamma + \gamma\alpha)\}$

$= (a)\{a^2 - 0\} = a^3$

7. (b) $\Delta = \begin{vmatrix} a & b & ax+b \\ b & c & bx+c \\ ax+b & bx+c & 0 \end{vmatrix} = 0$

$$\begin{vmatrix} 0 & 0 & ax^2+2bx+c \\ b & c & bx+c \\ ax+b & bx+c & 0 \end{vmatrix} = 0;$$

$R_1 \to R_1x + R_2 - R_3$

$(ax^2 + 2bx + c)(bx^2 + bc - acx - bc) = 0$

$(ax^2 + 2bx + c)(b^2x - acx) = 0$

$\Rightarrow b^2 - ac = 0$

$\Rightarrow b^2 = ac$

So, a, b, c are in GP

8. (b) $\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$

$$\begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} + \begin{vmatrix} x & x^2 & x^3 \\ y & y^2 & y^3 \\ z & z^2 & z^3 \end{vmatrix} = 0$$

$$\begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix}(1 + xyz) = 0 \Rightarrow xyz = -1$$

9. (b) $\begin{vmatrix} b+c & c+a & a+b \\ a+b & b+c & c+a \\ c+a & a+b & b+c \end{vmatrix} = k\begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}$

By $C_1 \to C_1 + C_2 + C_3$ on LHS det, we get

$$\Delta_1 = 2\begin{vmatrix} a+b+c & c+a & a+b \\ a+b+c & b+c & c+a \\ a+b+c & a+b & b+c \end{vmatrix}$$

$$= 2\begin{vmatrix} a+b+c & -b & -c \\ a+b+c & -a & -b \\ a+b+c & -c & -a \end{vmatrix}$$

$C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$

$$= 2\begin{vmatrix} a & -b & -c \\ c & -a & -b \\ b & -c & -a \end{vmatrix} C_1 \to C_1 + C_2 + C_3$$

$$= 2(-1)(-1)\begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix} = 2\begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}$$

Hence, $k = 2$

10. (b) $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 0$

$$\therefore\ \lambda = \begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix}$$

$$\lambda = abc\begin{vmatrix} \frac{1}{a}+1 & \frac{1}{b} & \frac{1}{c} \\ \frac{1}{a} & \frac{1}{b}+1 & \frac{1}{c} \\ \frac{1}{a} & \frac{1}{b} & \frac{1}{c}+1 \end{vmatrix}$$

$\{C_1 \to C_1 + C_2 + C_3\}$

$$= abc\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+1\right)\begin{vmatrix} 1 & \frac{1}{b} & \frac{1}{c} \\ 1 & \frac{1}{b}+1 & \frac{1}{c} \\ 1 & \frac{1}{b} & \frac{1}{c}+1 \end{vmatrix}$$

$R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$$= abc\begin{vmatrix} 1 & 1/b & 1/c \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} \quad \left\{\because \frac{1}{a}+\frac{1}{b}+\frac{1}{c}=0\right\}$$

$= abc$

11. (a) The system of equations
$x + y + z = 6,\ x + 2y + 3z = 10$
and $x + 2y + \lambda z = \mu$ has a unique solution

So, $\Delta \neq 0$

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & \lambda \end{vmatrix} \neq 0$$

$\lambda - 3 \neq 0$

$\Rightarrow \lambda \neq 3$ and $\mu \in R$

12. (b) The system of equations $x + y + z = 6$,
$x + 2y + 3z = 10$ and $x + 2y + \lambda z = \mu$ have no solution.
If $\Delta = 0$ and $\Delta_1, \Delta_2, \Delta_3$ are non-zero

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & \lambda \end{vmatrix} = 0 \Rightarrow \lambda = 3$$

$$\Delta_1 = \begin{vmatrix} 6 & 1 & 1 \\ 10 & 2 & 3 \\ \mu & 2 & 3 \end{vmatrix}$$

$$\Delta_2 = \begin{vmatrix} 1 & 6 & 1 \\ 1 & 10 & 3 \\ 1 & \mu & 3 \end{vmatrix} \text{ and } \Delta_3 = \begin{vmatrix} 1 & 1 & 6 \\ 1 & 2 & 10 \\ 1 & 2 & \mu \end{vmatrix}$$

When $\mu = 10$ then $\Delta_1, \Delta_2, \Delta_3$ are zero
So, for no solution $\mu \neq 0$
Hence, $\lambda = 3$ and $\mu \neq 10$

13. (a) Solve same as above question.
For inifnite soutions $\Delta = 0$ and $\Delta_1 = 0$
$\Rightarrow \lambda = 3$ and $\mu = 10$

14. (a) $\begin{vmatrix} bc & ca & ab \\ ca & ab & bc \\ ab & bc & ca \end{vmatrix} = 0$

Applying $C_1 \to C_1 + C_2 + C_3$

$$(ab + bc + ca)\begin{vmatrix} 1 & ca & ab \\ 1 & ab & bc \\ 1 & bc & ca \end{vmatrix} = 0$$

$\Rightarrow\ ab + bc + ca = 0$

$\Rightarrow\ \frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 0$

15. (c) $\begin{vmatrix} ka & k^2+a^2 & 1 \\ kb & k^2+b^2 & 1 \\ kc & k^2+c^2 & 1 \end{vmatrix} = \begin{vmatrix} ka & k^2 & 1 \\ kb & k^2 & 1 \\ kc & k^2 & 1 \end{vmatrix} + \begin{vmatrix} ka & a^2 & 1 \\ kb & b^2 & 1 \\ kc & c^2 & 1 \end{vmatrix}$

$$= k^3\begin{vmatrix} a & 1 & 1 \\ b & 1 & 1 \\ c & 1 & 1 \end{vmatrix} + k\begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix}$$

Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$$= 0 + k\begin{vmatrix} a & a^2 & 1 \\ -(a-b) & -(a^2-b^2) & 0 \\ c-a & (c^2-a^2) & 0 \end{vmatrix}$$

$$= k(a-b)(c-a)\begin{vmatrix} a & a^2 & 1 \\ -1 & -(a+b) & 0 \\ 1 & c+a & 0 \end{vmatrix}$$

$= k(a – b) (c – a) (– c – a + a + b)$
$= k(a – b) (b – c) (c – a)$

16. (b) The system of equations
$kx + 2y – z = 1, (k – 1) y – 2z = 2$
and $(k + 2)z = 3$ has a unique solution
So, $\Delta \neq 0$

$$\Rightarrow \begin{vmatrix} k & 2 & -1 \\ 0 & k-1 & -2 \\ 0 & 0 & k+2 \end{vmatrix} \neq 0$$

$\Rightarrow (k + 2) (k – 1) k \neq 0$
$\Rightarrow k \neq 0, k \neq 1, k \neq – 2$
So, $k = – 1$

17. (b) $$\Delta = \begin{vmatrix} 1 & \omega^3 & \omega^5 \\ \omega^3 & 1 & \omega^4 \\ \omega^5 & \omega^4 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 1 & \omega^2 \\ 1 & 1 & \omega \\ \omega^2 & \omega & 1 \end{vmatrix}$$

$$= \begin{vmatrix} 0 & 0 & \omega^2-\omega \\ 1 & 1 & \omega \\ \omega^2 & \omega & 1 \end{vmatrix} [R_1 \to R_1 - R_2]$$

$= (\omega^2 – \omega) \{\omega – \omega^2\}$
$= \omega^3 – \omega – \omega^2 + 1$
$= 2 – (\omega + \omega^2) = 2 – (– 1) = 3$

18. (c) The system of equations $x + ay + az = 0$, $bx + y + bz = 0$ and $cx + cy + z = 0$ has a nontrivial solution so

$$\Delta = 0 \Rightarrow \begin{vmatrix} 1 & a & a \\ b & 1 & b \\ c & c & 1 \end{vmatrix} = 0$$

$C_2 \to C_2 – C_1$ and $C_3 \to C_3 – C_1$

$$\begin{vmatrix} 1 & -1(1-a) & -(1-a) \\ b & 1-b & 0 \\ c & a & 1-c \end{vmatrix} = 0$$

$(1 – b)(1 – c) + b (1 – a) (1 – c) + c (1 – a)(1 – b) = 0$

$$\Rightarrow \frac{1}{1-a} + \frac{b}{1-b} + \frac{c}{1-c} = 0$$

$$\Rightarrow \frac{1}{1-a} - 1 + \frac{b}{1-b} + \frac{c}{1-c} = -1$$

$$\Rightarrow \frac{a}{1-a} + \frac{b}{1-b} + \frac{c}{1-c} = -1$$

19. (c) $$\Delta_1 = \begin{vmatrix} x & b & b \\ a & x & b \\ a & a & x \end{vmatrix}$$

$= x^3 – 3abx + ab^2 + a^2b$

$$\Delta_2 = \begin{vmatrix} x & b \\ a & x \end{vmatrix} = x^2 - ab$$

$$\frac{d}{dx}(\Delta_1) = \frac{d}{dx}(x^3 - 3abx + ab^2 + a^2b)$$

$= 3(x^2 – ab) = 3\Delta_2$

20. (d) $$\begin{vmatrix} p & b & c \\ a & q & c \\ a & b & r \end{vmatrix} = 0$$

$$\begin{vmatrix} p & b & c \\ -(p-a) & q-b & 0 \\ -(p-a) & 0 & r-c \end{vmatrix} = 0;$$

$R_2 \to R_2 – R_1, R_3 \to R_3 – R_1$
$\Rightarrow p(q – b) (r – c) + b(p – a) (r – c) + c(q – b) (p – a) = 0$

$$\Rightarrow \frac{p}{p-a} + \frac{b}{q-b} + \frac{c}{r-c} = 0$$

{On dividing by $(p – a) (q – b) (r – c)$}

$$\Rightarrow \frac{p}{p-a} + \left(\frac{b}{q-b}+1\right) + \left(\frac{c}{r-c}+1\right) = 2$$

$$\Rightarrow \frac{p}{p-a} + \frac{q}{q-b} + \frac{r}{r-c} = 2$$

21. (a) $$\begin{vmatrix} x & a & b \\ a & x & b \\ a & b & x \end{vmatrix} = 0;$$ Applying $C_1 \to C_1 + C_2 + C_3$

$$(x+a+b)\begin{vmatrix} 1 & a & b \\ 1 & x & b \\ 1 & b & x \end{vmatrix} = 0$$

$$(x+a+b)\begin{vmatrix} 1 & a & b \\ 0 & x-a & 0 \\ 0 & b-a & x-b \end{vmatrix} = 0$$

Applying $R_2 \to R_2 – R_1, R_3 \to R_3 – R_1$
$(x + a + b) (x – a) (x – b) = 0$
So, factors are $x – a, x – b$ and $x + a + b$

22. (d) $$\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix}$$

$$= \begin{vmatrix} 1+\omega+\omega^2 & 1+\omega+\omega^2 & 1+\omega+\omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix}$$

$= 0$ $\{\because 1 + \omega + \omega^2 = 0\}$

23. (b) $\Delta = \begin{vmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 1 & \log_y z \\ \log_z x & \log_z y & 1 \end{vmatrix}$

$\therefore \log_a b = \dfrac{\log b}{\log a}$

$$\Delta = \begin{vmatrix} 1 & \dfrac{\log y}{\log x} & \dfrac{\log z}{\log x} \\ \dfrac{\log x}{\log y} & 1 & \dfrac{\log z}{\log y} \\ \dfrac{\log x}{\log z} & \dfrac{\log y}{\log z} & 1 \end{vmatrix}$$

$$= \frac{1}{\log x \log y \log z}\begin{vmatrix} \log x & \log y & \log z \\ \log x & \log y & \log z \\ \log x & \log y & \log z \end{vmatrix} = 0$$

24. (b) $\begin{vmatrix} x-1 & 1 & 1 \\ 1 & x-1 & 1 \\ 1 & 1 & x-1 \end{vmatrix} = 0$

$(x-1)\{(x-1)^2 - 1\} - 1(x-1-1) + (1-x+1) = 0$
$x(x-1)(x-2) - (x-2) - (x-2) = 0$
$(x-2)\{x(x-1) - 2\} = 0$
$(x-2)(x+1)(x-2) = 0 \Rightarrow x = 2, -1$

25. (d) $\displaystyle\sum_{n=1}^{5} U_n = U_1 + U_2 + U_3 + U_4 + U_5$

Putting $n = 5$ in the formula for $\Sigma n, \Sigma n^2, \Sigma n^3$

$$U_n = \begin{vmatrix} 15 & 15 & 8 \\ 55 & 35 & 9 \\ 225 & 25 & 10 \end{vmatrix} = \begin{vmatrix} 0 & 15 & 8 \\ 20 & 35 & 9 \\ 200 & 25 & 10 \end{vmatrix} \{C_1 \to C_1 - C_2\}$$

$$= 20 \times 5 \begin{vmatrix} 0 & 3 & 8 \\ 1 & 7 & 9 \\ 10 & 5 & 10 \end{vmatrix}$$

$= 100\{-(30-40) + 10(27-56)\} = -2800$

26. (d) A is a $n \times n$ square matrix.
Then $|kA| = k^n |A|$

27. (b) $\Delta = \begin{vmatrix} 11 & 12 & 13 \\ 12 & 13 & 14 \\ 13 & 14 & 15 \end{vmatrix}$

Applying $C_3 \to C_3 - C_2$ and $C_2 \to C_2 - C_1$

$$\Delta = \begin{vmatrix} 11 & 1 & 1 \\ 12 & 1 & 1 \\ 13 & 1 & 1 \end{vmatrix} = 0$$

28. (d) Determinant Δ is of third order and every element is multiplied by 5.
Then new determinant $= 5^3\,\Delta = 125\,\Delta$

29. (c) $\begin{vmatrix} 3-x & -6 & 3 \\ -6 & 3-x & 3 \\ 3 & 3 & -6-x \end{vmatrix} = 0$

Applying $C_1 \to C_1 + C_2 + C_3$, we obtain

$$-x\begin{vmatrix} 1 & -6 & 3 \\ 1 & 3-x & 3 \\ 1 & 3 & -6-x \end{vmatrix} = 0$$

$$-x\begin{vmatrix} 1 & -6 & 3 \\ 0 & 9-x & 0 \\ 0 & 9 & -9-x \end{vmatrix} = 0$$

Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 \to R_1$
$\Rightarrow -x(9-x)(-9-x) = 0 \Rightarrow x = 0, 9, -9$

30. (a) The given system of equations $x + y + z = 2$, $2x + y - z = 3$ and $3x + 2y + kz = 4$ has a unique solution. So, $D \neq 0$

$$\begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 2 & k \end{vmatrix} \neq 0$$

$(k+2) - (2k+3) + (4-3) \neq 0$
$\Rightarrow k \neq 0$

31. (d) Let $\dfrac{1}{x} = X$ and $\dfrac{1}{y} = Y$

So, equations will become
$a_1X + b_1Y = c_1,\ a_2X + b_2Y = c_2$

So, $X = \dfrac{\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$ and $Y = \dfrac{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$

$X = -\dfrac{\Delta_2}{\Delta_1}$ and $Y = -\dfrac{\Delta_3}{\Delta_1}$

$\Rightarrow x = -\dfrac{\Delta_1}{\Delta_2}$ and $y = -\dfrac{\Delta_1}{\Delta_3}$

32. (d) $\begin{vmatrix} 1-i & \omega^2 & \omega \\ \omega^2 + i & \omega & -i \\ 1-2i-\omega^2 & \omega^2 - \omega & i - \omega \end{vmatrix}$

Applying $R_3 \to R_3 - (R_1 - R_2)$

$$\Delta = \begin{vmatrix} 1-i & \omega^2 & \omega \\ \omega^2 + i & \omega & -i \\ 0 & 0 & 0 \end{vmatrix} = 0$$

33. (c) $\Delta = \begin{vmatrix} 1 & bc & bc(b+c) \\ 1 & ca & ca(c+a) \\ 1 & ab & ab(a+b) \end{vmatrix} \{C_3 \to C_3 + abcC_1\}$

$$= \begin{vmatrix} 1 & bc & bc(a+b+c) \\ 1 & ca & ca(a+b+c) \\ 1 & ab & ab(a+b+c) \end{vmatrix}$$

$$= (a+b+c)\begin{vmatrix} 1 & bc & bc \\ 1 & ca & ca \\ 1 & ab & ab \end{vmatrix} = 0$$

or independent of a, b, and c

34. (a) $\Delta = \begin{vmatrix} i^n & i^{n+1} & i^{n+2} \\ i^{n+5} & i^{n+4} & i^{n+3} \\ i^{n+6} & i^{n+7} & i^{n+8} \end{vmatrix}$

$$= i^n \cdot i^{n+3} \cdot i^{n+6} \begin{vmatrix} 1 & i & i^2 \\ i^2 & i & 1 \\ 1 & i & i^2 \end{vmatrix} = 0$$

35. (d) $\Delta_2 = \begin{vmatrix} r & 2p & q \\ 2z & 4x & 2y \\ c & 2a & b \end{vmatrix} = 2\begin{vmatrix} r & p & q \\ 2z & 2x & 2y \\ c & a & b \end{vmatrix}$

$$\Delta_2 = 4\begin{vmatrix} r & p & q \\ z & x & y \\ c & a & b \end{vmatrix} = 2\begin{vmatrix} 2r & p & q \\ 2z & x & y \\ 2c & a & b \end{vmatrix} = 2\Delta_1$$

or $\dfrac{\Delta_1}{\Delta_2} = \dfrac{1}{2}$

36. (c) $\Delta = \begin{vmatrix} b(b-a) & b-c & c(b-a) \\ a(b-a) & a-b & b(b-a) \\ c(b-a) & c-a & a(b-a) \end{vmatrix}$

$$= (b-a)^2 \begin{vmatrix} b & b-c & c \\ a & a-b & b \\ c & c-a & a \end{vmatrix} \{C_2 \to C_2 + C_3\}$$

$$= (b-a)^2 \begin{vmatrix} b & b & c \\ a & a & b \\ c & c & a \end{vmatrix} = 0$$

37. (a) $\Delta = \begin{vmatrix} \log x & \log y & \log z \\ \log 2x & \log 2y & \log 2z \\ \log 3x & \log 3y & \log 3z \end{vmatrix}$

$$\Delta = \begin{vmatrix} \log x & \log y & \log z \\ \log 2 + \log x & \log 2 + \log y & \log 2 + \log z \\ \log 3 + \log x & \log 3 + \log y & \log 3 + \log y \end{vmatrix}$$

$\{R_3 \to R_3 - R_1, R_2 \to R_2 - R_1)$

$$= \begin{vmatrix} \log x & \log y & \log z \\ \log 2 & \log 2 & \log 2 \\ \log 3 & \log 3 & \log 3 \end{vmatrix} = 0$$

38. (b) $f(x) = \begin{vmatrix} a & -1 & 0 \\ ax & a & -1 \\ ax^2 & ax & a \end{vmatrix} = a\begin{vmatrix} 1 & -1 & 0 \\ x & a & -1 \\ x^2 & ax & a \end{vmatrix}$

$$= a\begin{vmatrix} 0 & -1 & 0 \\ x+a & a & -1 \\ x(x+a) & ax & a \end{vmatrix} \{C_1 \to C_1 + C_2\}$$

$f(x) = a(x+a)(a+x) = a(x+a)^2$
$f(2x) = a(2x+a)^2$
$f(2x) - f(x) = a\{(2x+a)^2 - (x+a)^2\}$
$= a(3x+2a)(x)$

39. (b) $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix}$

$\{R_2 \to R_2 - R_1, R_3 \to R_3 - R_1\}$

$$= \begin{vmatrix} 1 & 1 & 1 \\ 0 & x & 0 \\ 0 & 0 & y \end{vmatrix}$$

$D = xy$
It is divisible by both x and y

40. (c) $\Delta = \begin{vmatrix} \lambda+1 & \omega & \omega^2 \\ \omega & \lambda+\omega^2 & 1 \\ \omega^2 & 1 & \lambda+\omega \end{vmatrix}$

$R_1 \to R_1 + R_2 + R_3$
$\because\ 1 + \omega + \omega^2 = 0$

$$\therefore\ \Delta = \lambda\begin{vmatrix} 1 & 1 & 1 \\ \omega & \lambda+\omega^2 & 1 \\ \omega^2 & 1 & \lambda+\omega \end{vmatrix}$$

$$= \lambda\begin{vmatrix} 1 & 0 & 1 \\ \omega & \lambda+\omega^2-\omega & 1-\omega \\ \omega^2 & 1-\omega^2 & \lambda+\omega-\omega^2 \end{vmatrix}$$

$\{C_1 \to C_2 - C_1, C_3 \to C_3 - C_1\}$
$= \lambda\{\lambda^2 - \omega^4 - \omega^2 + 2 - 2 + \omega + \omega^2\} = \lambda^3$

41. (c) $\Delta = \begin{vmatrix} {}^8C_3 & {}^9C_5 & {}^{10}C_7 \\ {}^8C_4 & {}^9C_6 & {}^{10}C_8 \\ {}^9C_n & {}^{10}C_{n+2} & {}^{11}C_{n+4} \end{vmatrix} = 0$

$R_1 \to R_1 + R_2$
$\{{}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r\}$

$$\Rightarrow \begin{vmatrix} {}^9C_4 & {}^{10}C_6 & {}^{11}C_8 \\ {}^8C_4 & {}^9C_6 & {}^{10}C_8 \\ {}^9C_n & {}^{10}C_{n+2} & {}^{11}C_{n+4} \end{vmatrix} = 0$$

$\Rightarrow n = 4.$

42. (d) $\Delta = \begin{vmatrix} 1 & 1-i & -i \\ 1+i+\omega^2 & -1 & -1+\omega-i \\ \omega^2 & \omega^2-1 & \omega-1 \end{vmatrix}$

$C_2 \to C_2 - (C_1 + C_3)$

$= \begin{vmatrix} 1 & 0 & -i \\ 1+i+\omega^2 & 0 & -1+\omega-i \\ \omega^2 & 0 & \omega-1 \end{vmatrix} = 0$

43. (b) $\Delta = \begin{vmatrix} a^2+b^2 & c^2 & c^2 \\ a^2 & b^2+c^2 & a^2 \\ b^2 & b^2 & c^2+a^2 \end{vmatrix}$

$\because$ a, b, c are cube roots of unity

So, $a = 1, b = \omega, c = \omega^2$

$\Rightarrow \Delta = \begin{vmatrix} 1+\omega^2 & \omega^2 & \omega \\ 1 & \omega^2+\omega & 1 \\ \omega^2 & \omega^2 & \omega+1 \end{vmatrix} \quad \{C_1 \to C_1 + C_2\}$

$\because 1 + \omega + \omega^2 = 0$

$\Delta = \begin{vmatrix} 0 & \omega^2 & \omega \\ 0 & \omega^2+\omega & 1 \\ 2\omega^2 & \omega^2 & \omega+1 \end{vmatrix}$

$= 2\omega^2(\omega - 1 - \omega^2) = 2\omega^2(\omega + \omega)$

$\Rightarrow \Delta = 4\omega^3 = 4$

$\Rightarrow \text{Im}(\Delta) = 0$

44. (b) $\Delta = \begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix}$

$\{C_1 \to C_1 + C_2 + C_3\}$

$\Delta = 2(a+b+c)\begin{vmatrix} 1 & a & b \\ 1 & b+c+2a & b \\ 1 & a & c+a+2b \end{vmatrix}$

$\begin{cases} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{cases}$

$= 2(a+b+c)\begin{vmatrix} 1 & a & b \\ 0 & b+c+a & 0 \\ 0 & 0 & c+a+b \end{vmatrix}$

$= 2(a+b+c)^3\begin{vmatrix} 1 & a & b \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$

$= 2(a + b + c)^3$

45. (c) $\Delta = \begin{vmatrix} a & a+b & a+b+c \\ 3a & 4a+3b & 5a+4b+3c \\ 6a & 9a+6b & 11a+9b+6c \end{vmatrix}$;

$\{R_3 \to R_3 - 2R_2, R_2 \to R_2 - 3R_1\}$

$= \begin{vmatrix} a & a+b & a+b+c \\ 0 & a & 2a+b \\ 0 & a & a+b \end{vmatrix}$

$= a\{a^2 + ab - 2a^2 - ab\}$

$= -a^3 = -(\text{i})^3$

$= i$

46. (c) $\begin{vmatrix} p & q-y & r-z \\ p-x & q & r-z \\ p-x & q-y & r \end{vmatrix} = 0$

or $xyz\begin{vmatrix} \frac{p}{x} & \frac{q}{y}-1 & \frac{r}{z}-1 \\ \frac{p}{x}-1 & \frac{q}{y} & \frac{r}{z}-1 \\ \frac{p}{x}-1 & \frac{q}{y}-1 & \frac{r}{z} \end{vmatrix} = 0$

$\{C_1 \to C_1 + C_2 + C_3\}$

$\Rightarrow \left(\frac{p}{x}+\frac{q}{y}+\frac{r}{z}-2\right)\begin{vmatrix} 1 & \frac{q}{y}-1 & \frac{r}{z}-1 \\ 1 & \frac{q}{y} & \frac{r}{z}-1 \\ 1 & \frac{q}{y}-1 & \frac{r}{z} \end{vmatrix} = 0$

$\{R_2 \to R_2 - R_1, R_3 \to R_3 - R_1\}$

$\left\{\frac{p}{x}+\frac{q}{y}+\frac{r}{z}-2\right\}\begin{vmatrix} 1 & \frac{q}{y}-1 & \frac{r}{z}-1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 0$

$\Rightarrow \frac{p}{x}+\frac{q}{y}+\frac{r}{z}-2 = 0$

or $\frac{p}{x}+\frac{q}{y}+\frac{r}{z} = 2$

47. (b) Let $\Delta = \begin{vmatrix} \cos(\theta+\phi) & -\sin(\theta+\phi) & \cos 2\phi \\ \sin\theta & \cos\theta & \sin\phi \\ -\cos\theta & \sin\theta & \cos\phi \end{vmatrix}$

$= \cos(\theta + \phi)\{\cos\theta \cos\phi - \sin\theta \sin\phi\}$
$+ \sin(\theta + \phi)\{\sin\theta \cos\phi + \cos\theta \sin\phi\}$
$+ \cos 2\phi\{\sin^2\theta + \cos^2\theta\}$

$= \cos^2(\theta + \phi) + \sin^2(\theta + \phi) + \cos 2\phi$

$= 1 + \cos 2\phi = 2\cos^2\phi$

i.e., Δ is independent of θ

48. (b) $\begin{vmatrix} x+a & a^2 & a^3 \\ x+b & b^2 & b^3 \\ x+c & c^2 & c^3 \end{vmatrix} = 0$

$$x\begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix} + abc\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = 0$$

$$x\begin{vmatrix} 1 & a^2 & a^3 \\ 0 & b^2-a^2 & b^3-a^3 \\ 0 & c^2-a^2 & c^3-a^3 \end{vmatrix} + abc\begin{vmatrix} 1 & a & a^2 \\ 0 & b-a & b^2-a^2 \\ 0 & c-a & c^2-a^2 \end{vmatrix} = 0$$

$\{R_2 \to R_2 - R_1, R_3 \to R_3 - R_1\}$

$$\left\{x\begin{vmatrix} 1 & a^2 & a^3 \\ 0 & b+a & b^2+a^2+ab \\ 0 & c+a & c^2+a^2+ac \end{vmatrix} + abc\begin{vmatrix} 1 & a & a^2 \\ 0 & 1 & b+a \\ 0 & 1 & c+a \end{vmatrix}\right\} = 0$$

$\{R_3 \to R_3 - R_2\}$

$$\left\{x\begin{vmatrix} 1 & a^2 & a^3 \\ 0 & b+a & b^2+a^2+ab \\ 0 & c+a & c+b+ac \end{vmatrix} + abc\begin{vmatrix} 1 & a & a^2 \\ 0 & 1 & b+a \\ 0 & 0 & 1 \end{vmatrix}\right\} = 0$$

$x\{ab + bc + ca\} + abc = 0$

$$x = \frac{-abc}{ab+bc+ca} = -\frac{abc}{\sum ab}$$

49. (d) $\Delta = \begin{vmatrix} x^2 & x^2-(y-z)^2 & yz \\ y^2 & y^2-(z-x)^2 & zx \\ z^2 & z^2-(x-y)^2 & xy \end{vmatrix}$

$$= \begin{vmatrix} x^2 & x^2 & yz \\ y^2 & y^2 & zx \\ z^2 & z^2 & xy \end{vmatrix} - \begin{vmatrix} x^2 & (y-z)^2 & yz \\ y^2 & (z-x)^2 & zx \\ z^2 & (x-y)^2 & xy \end{vmatrix}$$

$$= 0 - \begin{vmatrix} x^2 & y^2+z^2 & yz \\ y^2 & z^2+x^2 & zx \\ z^2 & x^2+y^2 & xy \end{vmatrix} \quad \{C_2 \to C_2 + 2C_3\}$$

$$= -(x+y^2+z^2)\begin{vmatrix} x^2 & 1 & yz \\ y^2 & 1 & zx \\ z^2 & 1 & xy \end{vmatrix} \quad \{C_2 \to C_2 + C_1\}$$

$$= -(x^2+y+z^2)\begin{vmatrix} x^2 & 1 & yz \\ y^2-x^2 & 0 & z(x-y) \\ z^2-x^2 & 0 & y(x-z) \end{vmatrix}$$

$\{R_2 \to R_2 - R_1, R_3 \to R_3 - R_1\}$

$= -(x^2+y^2+z^2)(y-z)(z-x)$

$$\begin{vmatrix} x^2 & 1 & yz \\ y+x & 0 & -z \\ z+x & 0 & -y \end{vmatrix} \quad \{R_3 \to R_3 - R_2\}$$

$= -(x^2+y^2+z^2)(y-x)(z-x)(z-y)$

$$\begin{vmatrix} x^2 & 1 & yz \\ y+x & 0 & -z \\ 1 & 0 & 1 \end{vmatrix}$$

$= (x-y)(y-x)(z-x)(x+y+z)(x^2+y^2+z^2)$

50. (c) Equations have non-zero solution

so $\begin{vmatrix} a^3 & (a+1)^3 & (a+2)^3 \\ a & a+1 & a+2 \\ 1 & 0 & 1 \end{vmatrix} = 0$

By hit and trial method

$a = -1$